

This puzzle uses triple homophones. The 'strange sort of sense' is weakly clued by the fact that the next sentence of the flavor text uses "they're", "their" and "there".

There are a few cases (like 'small child' + 'under high tension') where the spacing in the middle means neither side's answer fits, which is a good place to cue the need for a triplet. The number of spaces in shift-6's middle rules out 'carrot' as an answer

The Temple of Apollo at Delphi

Archaeologists propose that the prophecies of the oracles in the Temple of Apollo were powered by toxic fumes from naturally-occurring fissures. It may have been crazy ranting in Greek, but in English, the following gibberish makes a strange sort of sense. They're missing their middle pieces, but maybe the clues can tell you what belongs there.

heeled (SOLED)≠
board for a painter (PALETTE)≠
hunts for food (PREYS)≠
referenced (CITED)≠
 small child (TOT)≠
work dough (KNEAD)≠
striped asphalt strip (ROAD)≠
seasonal illness (FLU)≠
fruity drink suffix (ADE)≠
 shift-6 (CARET)≠
not a vegetarian dish (MEAT)≠
sovereignty (REIGN)≠
turn solid (FREEZE)≠
trim down (PARE)≠
black out words (CENSOR)≠
ritual acts (RITES)≠

SOLD (3,1) ≠ (SOULED) imbued with a spirit
PALATE (6,3) ≠ (PALLET) wooden shipping platform
PRAISE (3,4) ≠ (PRAYS) communicates to a deity
SIGHTED (3,5) ≠ (SITED) located
TAUGHT (3,6) ≠ (TAUT) under high tension
NEED (2,3) ≠ (KNEED) struck with a leg joint
RODE (2,3) ≠ (ROWED) propelled by hand
FLEW (1,2) ≠ (FLUE) smoke escape route
 AID (1,2) ≠ (AIDE) assistant
KARAT (3,1) ≠ (CARAT) weight of gems but conspicuously not gold
METE (3,2) ≠ (MEET) to come together
RAIN (3,2) ≠ (REIN) animal guidance rope
FREES (5,1) ≠ (FRIEZE) decorative band beneath a cornice
PAIR (2,3) ≠ (PEAR) pyriform fruit
SENSOR (3,4) ≠ (CENSER) ceremonial burner
RIGHTS (6,4) ≠ (WRITES) composes text

The first of each pair of numbers indexes into the middle word to read "LEAGUE OF AR T I S A N S" and the second of each pair makes "SLITTED LIKE A F I S H". These clue GUILD and GILLED, which have a triplet: GILD, which is the answer.